The Physics of Lift in Hot-Air Ballooning

by William G. Phillips

I just recently began flying balloons, and typical of any new student, I have besieged my poor instructor with an unrelentless chain of questions. Many of the things I read just don't make sense to me. One prime example is the statement "1000 cubic feet of hot air will lift 17 pounds, while helium lifts 60 pounds." This is just not so! Every self-respecting balloonist knows that the same amount of hot air on a cold day at sea level packs a bigger load than it will in warmer conditions at 5000 feet. To prove my point, late one night I sat down in front of my personal computer to accurately calculate just what the lift is for different balloons at different altitudes (pressures), outside air temperatures (OAT), and internal envelope temperatures (T_i).

The next few paragraphs of this paper will lead you through the physics and derivation of a formula. That formula will provide a tool to calculate the effective payload (passenger and carry-on items like champagne) of any balloon for *any* given OAT and altitude. You can use the formula to generate performance curves for your own balloon.

Part 1

Before we get involved in the algebra, let us look at the nature of how balloons fly. The physics of lift from balloons is a direct application of Archimedes principle. Good old Archimedes said that the buoyant force exerted on any object floating in a fluid is equal to the weight of the fluid that the object displaces. That's why heavy steel ships float in the ocean. They displace a volume of ocean water which is EXACTLY equal to their weight. If you load more people on the ship, it sinks deeper, displacing more water until the weight of the displaced water again equals the entire weight of the ship. Well, a balloon does the same thing. When you fill an AX-7 full of cold air, it displaces about 75,000 cubic feet (ft^3) of air which weighs around 6000 pounds at sea level (1 atmosphere of pressure) and 60° Fahrenheit (F) OAT. This means that you have 6000 pounds of lift as soon as you cold inflate! The trouble is, you added 6000 pounds of cold air to the weight of your balloon in the process! So, your effective lift (useful load) is zero. Now, if you heat that internal air it becomes less dense (less mass per ft^3) and the entire weight of the internal bubble of air decreases. If you keep on heating the bubble up to about 200° F its weight reduces to about 4700 pounds.

So let's add up the total weight of the balloon at this point.

1) Balloon, i.e. envelope, burners, basket, fuel tank, champagne, strikers, etc. weighs about 730 pounds for an AX-7

2) Pilot and passenger weigh about 300 pounds

3) Hot air in the envelope weighs about 4700 pounds

The sum of these items is 5730 pounds. If we subtract this from our buoyant force (6000 pounds) we find about 270 pounds of residual lift. This will produce a nice climb, or we can interpret this result as the number of additional pounds of payload we can carry and still maintain neutral buoyancy (no climb, no descent). From this calculation we can also see that there will be no trouble carrying this load with a T_i less than 200° F. In fact, neutral buoyancy for these conditions will occur at a T_i of about 175° F.

Now, have I sparked your interest? How do I know, or how can I anticipate what envelope temperature will be required for a given load and given atmospheric conditions? Therein lies the beauty of science and the language of mathematics to predict how something will behave. It actually turns out that these numbers are quite easy to calculate once you have the formula. When you perform your own calculations the results can be verified very accurately upon testing your balloon. If you are impatient with arithmetic just skip over to **Part 2** where you see the symbolic mumbo-jumbo begin to subside. For those of you who enjoy the mathematics, here's the BEEF!

To start, we need the laws generated by a couple of good old boys named Charles and Boyles. They came up with a formula like this:

$$PV=nRT \quad (1)$$

This says that the product of a gas volume (V) and its pressure (P) is directly proportional to the amount of gas we have (n) and its absolute temperature (T). R is simply a proportionality constant (a number) which depends upon which system of units one prefers to work with, i.e. metric, English, or whatever.

What we need to do is use equation (1) to relate the atmospheric conditions and T_i (internal envelope temperature of our balloon) to the total weight of the displaced air and the total weight of the hot air. Once we have these we can subtract the two (total lift), then subtract the balloon weight and BINGO! We have "PAYLOAD." In equation form, this looks like:

Payload = $(n_o - n_i)g - wt_b$ (2)

Where:

n_o = (n outside) is the number of moles of the cold displaced air

n_i = (n inside) is the number of moles of the hot internal air

g = the number of grams of air per mole

wt_b = the weight of the balloon

From equation (1) we can solve for the n's in terms of pressure, volume and temperature,

$n_o = P_oV_o/RT_o$ and $n_i = P_iV_i/RT_i$ (3)

where T_o is the absolute cold air temperature and T_i is the absolute hot air temperature. Substituting these expressions into equation (2), we get

$$\text{Payload} = (P_oV_o/RT_o - P_iV_i/RT_i)g - wt_b \quad (4)$$

Since the internal pressure of a hot-air balloon is the same as the outside ambient pressure we let $P_i=P_o=P$. Likewise, the volume of displaced air outside of the balloon equals the volume of the hot air inside the balloon so, $V_i=V_o=V$.

Rewriting (4),

$$\text{Payload} = (PV/RT_o - PV/RT_i)g - wt_b$$

and factoring out PV/R

$$\text{Payload} = PV/R\ (1/T_o - 1/T_i)g - wt_b$$

moving **g** to the front of the equation and rewriting a little gives us,

$$\text{Payload} = PV_g/R\ (1/T_o - 1/T_i) - wt_b$$

This is almost a workable formula but we need to figure out what values to use for **g** and **R** so we can plug in numbers and get answers in the English system, i.e. cubic feet (ft^3) for volume, atmospheres for pressure and pounds for weight. Quickly, for you purists let's assign some numbers to all of these symbols so we can solve real problems. Let

g=28.8 gm/mole

for an atmospheric gas ration assuming 80% nitrogen and 20% oxygen (quick and dirty, but accurate to 1 or 2%). The ideal gas constant is:

$R = 2.90 \times 10^{-3}$ atm ft^3/mole K°

We easily find the following ratio.

g/R = 9943 gm K°/atm ft^3

To convert to the English system we know that there are 454 grams per pound. So:

g/R = 9943/454 = 21.9 lb K°/atm ft^3

Finally,

Payload = $PV(1/T_o - 1/T_i)$ * (21.9 lb K°/atm ft^3) - wt_b (6)

(* used to indicate multiplication)

Now what this equation will give us is the amount of weight we can carry for a given atmospheric pressure (P), i.e. altitude, ambient temperature (T), and whatever we feel comfortable with as an internal envelope temperature (T_i).

There's only one more catch to using equation (6) and that lies in the fact that T_i and T_o must be converted to degrees Kelvin (an absolute temperature scale where 0 degrees means the temperature where all molecular motion stops). Figure 1 is a chart which will allow you to convert degrees F to degrees K. We also need to convert altitude to pressure in atmospheres (atm) and Figure 2 gives you a nice chart for performing that task.

Part 2

Now let's work a problem so you can start using this formula for your own set of conditions.

Conditions:

Ambient Temperature - 60° F (T_o)

Balloon weight - 730 pounds (Wt)

Balloon volume - 77,400 ft^3 (V)

Internal envelope temperature - 200° F (T_i)

Altitude (5000 ft) - .85 atm (P)

First we have to convert 60° F and 200° F into Kelvin temperatures. Looking at Figure 1 we get T_i = 288.5° K and T_o = 366.3° K. So our list now looks like:

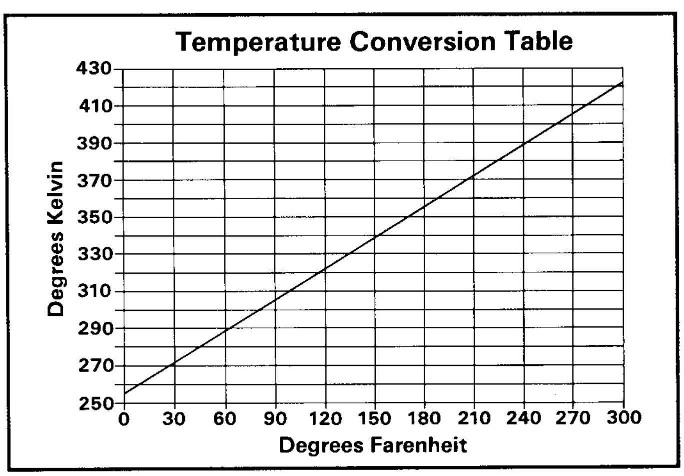

Figure 1

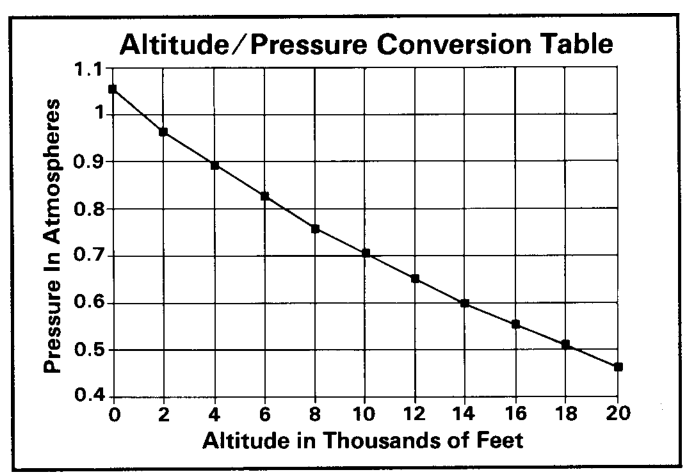

Figure 2

$T_i = 288.5°$ K
$T_o = 366.3°$ K
P = .85 atm
$V = 77{,}400\ ft^3$
$wt_b = 730$ pounds

Now let just plug these numbers into equation (6) and find out how much weight we can carry:

Payload=[.85 * 77,400 * (1/288.5 - 1/366.3) * 21.9] - 730
Payload = 331 pounds

This means that you can carry up to 331 pounds of people, champagne, strikers, lighters, gum, etc. on your balloon flight and have neutral weight at an internal air temperature of 200° F. Now if you would like to go up to 10,000 ft (P = .7 atm) then you need to solve the equation again for the higher conditions (reduced pressure and temperature). Lets assume the temperature drops 15° F going to 10,000 ft and we'll crank out the answer.

$T_o = 45°$ F = 280.2° K
$T_i = 200°$ F = 366.3° K
P = .7 atm
$V = 77{,}400\ ft^3$
$wt_b = 730$ pounds

The maximum payload at 10,000 ft is then
Payload = [.7 * 77,400 * (1/280.2 - 1/366.3) * 21.0] - 730 pounds
Payload = 265 lbs

So you can see that the maximum weight you may carry for a given internal envelope temperature decreases with altitude. Even though the ambient temperature goes down (which increases lift) the atmospheric pressure drop is a more dominant effect.

I had a lot of fun with this formula. In fact from it I derived another formula to find out what my internal temperature T_i would have to be to carry a given payload to a specific altitude and OAT. I use this formula when two or three of us want to fly high but maintain a reasonable T_i. Without a lot of derivation one can easily turn equation (6) into

$T_i = (T_o PV * 21.9)/(1 - T_o(\text{payload} + wt_b))$ (7)

I list this equation for those of you who may want to program a home computer to grind out the results of your particular balloon for many different conditions.

Some of the more technically inclined readers are probably wondering why I allowed myself to assume a uniform internal envelope temperature T_i. You know, of course, it isn't really right because the air at the crown is hotter than at the throat. Consider, however, that 80 to 90 percent of the hot air mass (of most balloon designs) resides in the top 2/3 of the height of the envelope. In this region there is a lot of turbulent flow going on, especially when pilots use short blasts of heat in repetition. So I just assumed uniform mixing and therefore a uniform heat distribution inside.

This formula (which has derived from ideal gas laws) may give you a slight variation in payload when you test it against the actual performance of your balloon. You can, however, adjust it for your balloon by inserting a small correction factor. For instance assume that for your balloon you consistently find that you get only 95% of the load the formula says you should get. In this case always "Tweek" the formula by multiplying the result of equation (6) by .95 (the fudge-factor).

The last figure (3) is a performance chart I generated on my computer. I wrote a program to do these charts for any balloon at any altitude. It just uses equation (6) for various ambient temperatures (the diagonal lines), and envelope temperatures (across the bottom axis of the chart). Here the weight, volume and altitude of the balloon are fixed. To read the chart simply pick the payload you would like to carry along the left vertical side. Then move horizontally to the diagonal depicting the ambient temperature. The position straight down from that point tells you how hot you have to burn to lift that weight.

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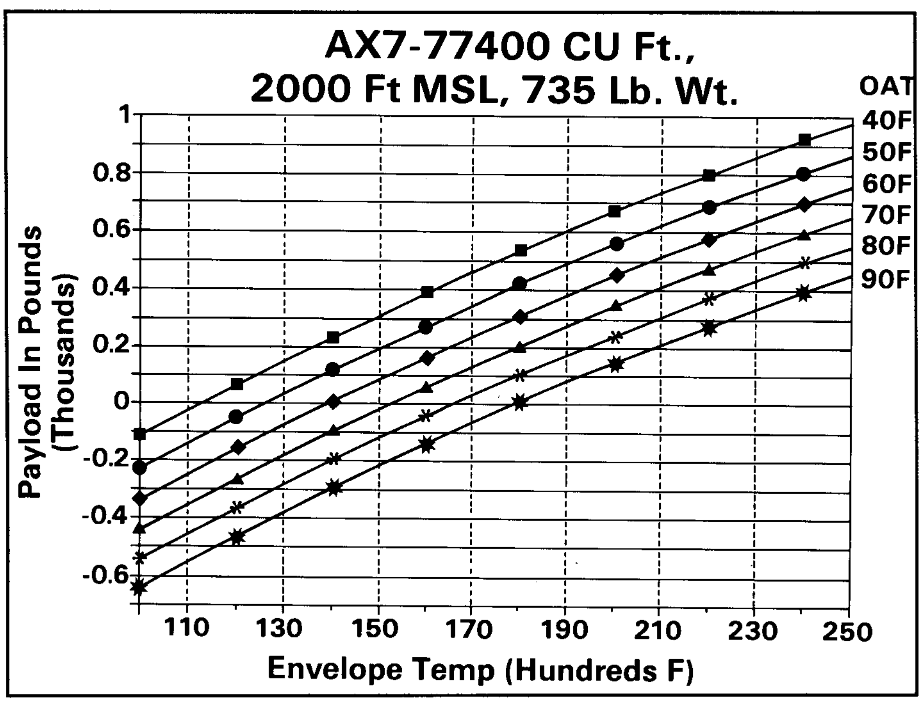


Figure 3 Performance Chart

The Forces and Pressures of Balloon Flight

by William G. Phillips

Did you ever wonder what the forces inside of your balloon were like? Sure, we all know that hot air rises and if you capture hot air in bag, the bag goes up with it. People talk about pressures, forces, loads, strength of materials, etc. but on a detailed level, what really is happening to make your balloon fly? If balloons have pressure in them, then why don't they deflate themselves like a small rubber blow-up balloon? Since we don't tie off our balloon throats why doesn't the high pressure air escape?

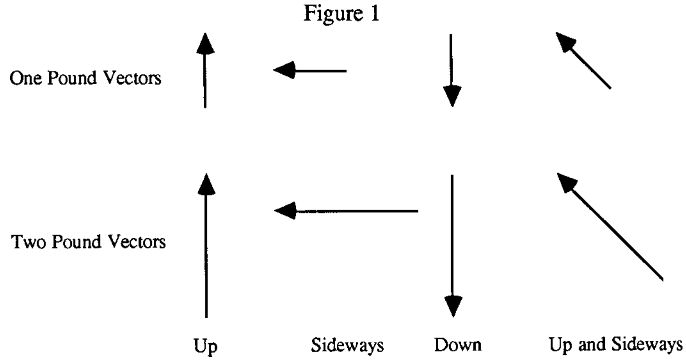
It's sort of obvious that there's pressure in a hot air balloon. Just pull on the vent and feel the resistance. If you pack more people into the gondola and a few more bottles of champagne, the vent is harder to work! That means that the pressure went up. The higher pressure applies higher force on the vent to keep it sealed. This pressure in the balloon can't be distributed uniformly over the entire envelope. If it were, there would be a pressure differential between the throat and the atmosphere. In that case all of the air would come roaring out of the throat and deflate the balloon. An additional clue to the nonuniformity in pressure can be gleaned from the fight manual. You're allowed good sized tears near the throat but only tiny ones above the equator. That's because the loss of hot air is greater for the same sized hole the higher you go in the balloon.

A Few Definitions

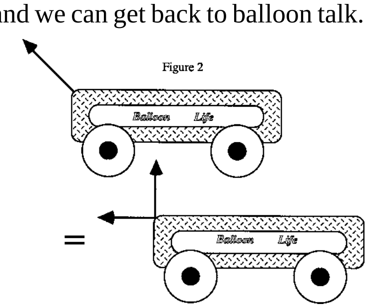
If we're going to talk about pressure we all need to agree on *how* we talk about it. Pressure is the measure of force exerted on a given area. If I apply a *force* of 10 pounds uniformly over a square foot of cloth, then we say the *pressure* is 10 pounds per square foot. If I distribute the same force over 2 square feet then we could say we have a pressure of 10 pounds on 2 square feet or simply 5 pounds per square foot. So pressure is the force exerted on something divided by the area of that something, i.e. force per unit area.

Well, all that is nice but what is *force*? Everyone probably thinks that they already know what force is, and maybe they do, but we should define a couple of things anyway. If you hold onto a bottle of champagne it applies a force to your hand of a couple pounds-toward the earth. If you strap a bunch of helium-filled balloons to your wrist you might get 2 pounds force again but-toward the sky. So when we talk about force, we always need to talk about direction too! We conveniently do this on paper by drawing arrows (called vectors). (Figure 1) When we draw these arrows their length represents the amount of their magnitude. Their direction, of course, represents their direction. For example a 2-pound upward force is drawn twice as long as a 1-pound upward force, a 3-pound force 3 times longer, and so on.

Figure 1



Only one more concept and we can get back to balloon talk. This concerns the up and sideways vector. If I pull a kid's wagon with an up and sideways force, I can accomplish the same thing by making 2 separate forces at 90 degrees to each other which are equivalent. (Figure 2)



In the second part of the picture I have reduced the up and sideways force to 2 equivalent forces called components and the kid in the wagon will never know the difference. Any force can be replaced by substituting other equivalent forces for it. This is a powerful concept because it allows us to really see along which directions these forces are pushing things. Now, since pressure is force per unit area and force has two intrinsic attributes, i.e. direction and magnitude, pressure also has direction and magnitude.

About Pressure

Anyone who has ever snorkeled or SCUBA dived knows that you have to clear your ears as you go down to greater depths.

That’s because the water pressure increases with depth. Since the water pressure doubles if you double your depth and triples if you triple your depth, we say that the pressure increases linearly. What this really means is that if you draw a graph of pressure vs. depth, you get a straight line (Figure 3). Now lets fill a can with water and look at the pressure differential (Figure 4) between the inside of the can and the outside world (Throughout most of the remainder of this article I will use the term “Pressure” to imply “Pressure Differential”). The pressure is zero just at the surface “A”. The pressure at “B” is an amount half way between zero and “C” and the pressure at “C” is twice that at “B”. Let’s reshape our can a little bit and fill it again with water (Figure 5). The pressure at “A” is zero, at “B” is 1/2 “C” and at “C” is twice “B” again.

Looking at figures 4 and 5, we notice a couple of things. First, the shape of the can has nothing to do with the increase in pressure with depth! This makes sense because you will experience the same pressure under 10 feet of water whether it’s in a small pool, a big pool, or in the ocean. Secondly, the pressure at the top of the can is the same as that of the outside environment. Hence, although the pressure increases as we go down in the can, there is no force trying to throw the water out the top of the can because we’re back to neutral at the top.

Let’s take the same bulbous shaped can and fill it with cold dense air. Well, it behaves about the same as it did with water. The pressure produced by the weight of those cold dense molecules of air pushing down on one another increases with depth. If we try to draw the pressure and associated vectors in one picture they look something like figure 6. Both the pressure and associated force vectors start small at the top and get larger toward the bottom. It’s because of these forces that the can weighs more, full of cold air, than it would with normal temperature air in it. The areas of downward forces have effectively increased the weight!

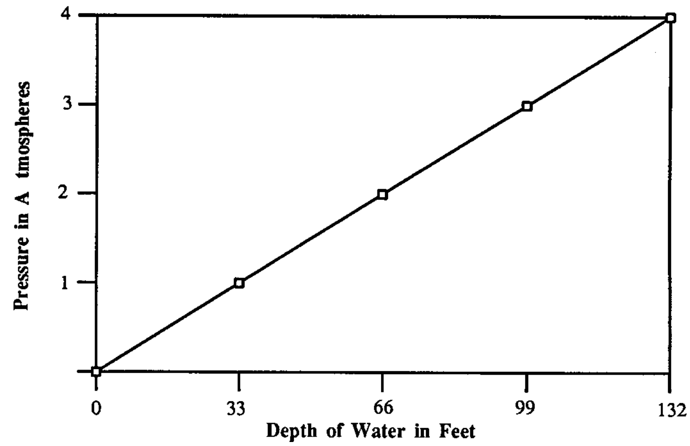
Well, you can see where all of this is leading. Let’s turn the can around fill it with hot not-very-dense air and we’ll get a similar pressure distribution (Figure 7). Here again, the pressure at the throat is zero and increases as you go up toward the crown due to all of the hot little molecules pushing up on each other. Unlike water, however, air is somewhat compressible. You don’t quite get nice linear behavior in pressure with height above the throat (For you purists, the pressure differential gradient in a balloon is exponential just like the Earth’s atmosphere). For our purposes though, we will assume it to be linear. This will work well enough to give us some nice approximations.

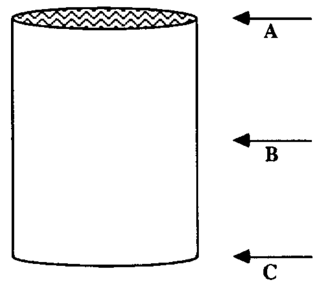
Lift

When I first drew the picture in figure 7, I noticed that all pressure vectors below the equator have downward components. What this means is that all pressure pushing on the cloth below the equator is effectively increasing the weight of the balloon. Only the forces above the equator help lift.

One lonely night I decided to crank up my computer and grind out just what was happening. Since the balloons we fly are practically spherical, I started with a perfectly spherical shape (the spherical shape makes the math **ALOT** easier!). I also assumed a linear increase in pressure from the throat to the crown. To begin with, I calculated the sums of the vertical components

Figure 3 Linear Pressure Curve with Water Depth





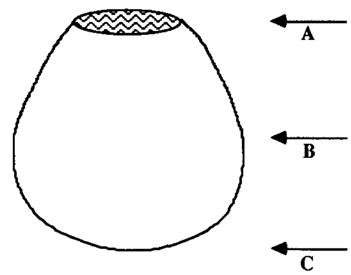


Figure 4

Figure 5

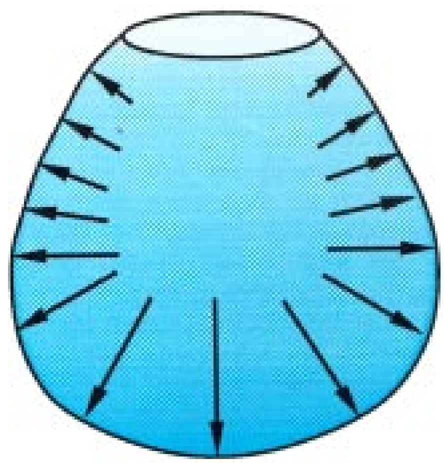


Figure 6

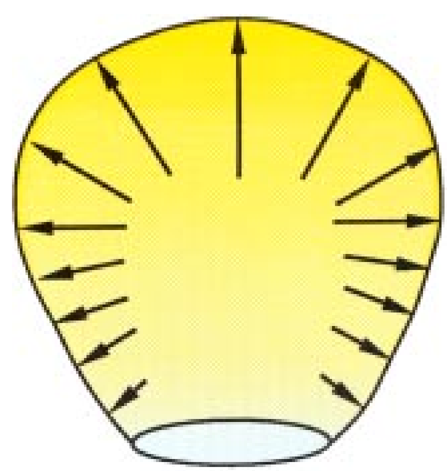


Figure 7

of force at each level of the envelope. This was accomplished by breaking the sphere up into many narrow bands similar to the 2 shown in figure 8 and using a little trigonometry.

Notice figure 9 that, because the balloon is symmetrical, each horizontal vector component has an exact opposite on the other side of the envelope. Thus, all of these horizontal forces effectively cancel each other. This is intuitively reasonable since an inflated balloon has no tendency to want to move sideways on its own! The vertical components, however, are larger in the upper portion of the balloon thus outweighing their negative lower counterparts. This antisymmetric property is what produces our lift. The actual vertical force on each of these bands can be calculated by simply multiplying the vertical component of pressure (force per unit area) by the surface area of the band (area).

Assuming a 60-foot diameter balloon and subdividing the envelope into 60 narrow bands (one for each foot of height), I calculated the force on each, and produced the following plot (Figure 10). This curve is based on lifting a 1000-pound gross load. The horizontal axis of the plot is a measure of the height above the throat (the throat was assumed to be the bottom point of the sphere). The vertical axis is the total lifting force in pounds produced by that particular band on the envelope.

An interesting point of geometry is that each band has the same surface area although the bands vary in shape and size (188 square feet). In fact at the crown the band becomes a cap just like a parachute top. The proof of this is a little complicated

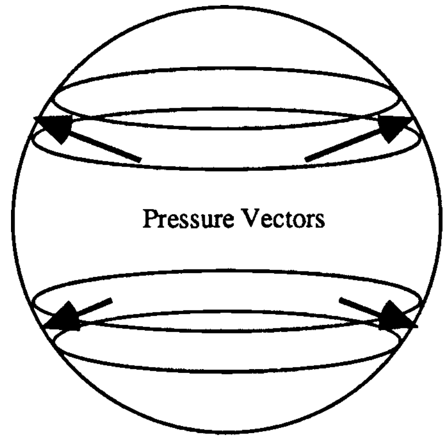
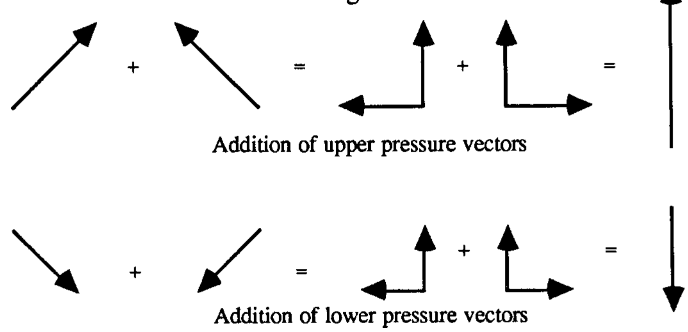
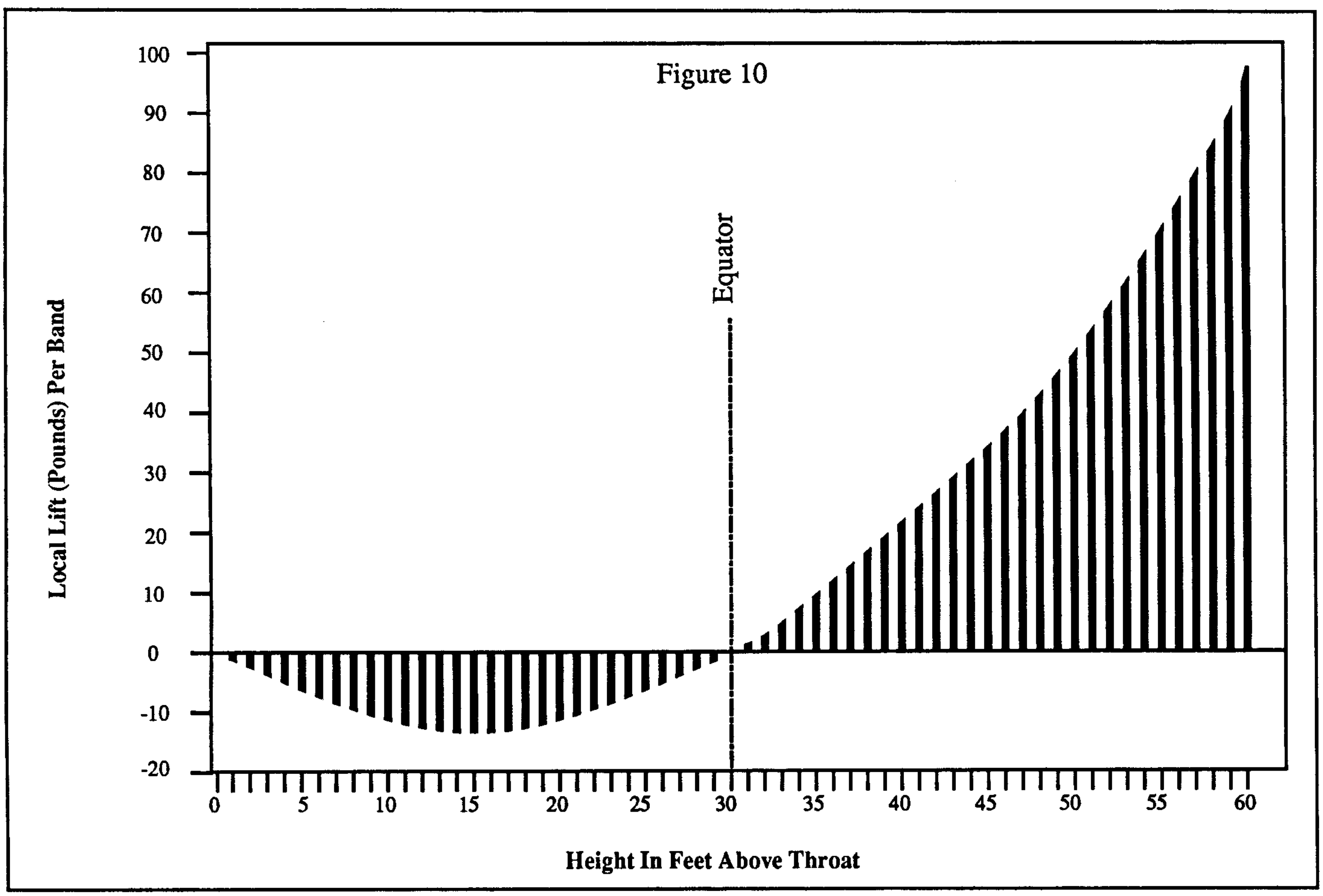
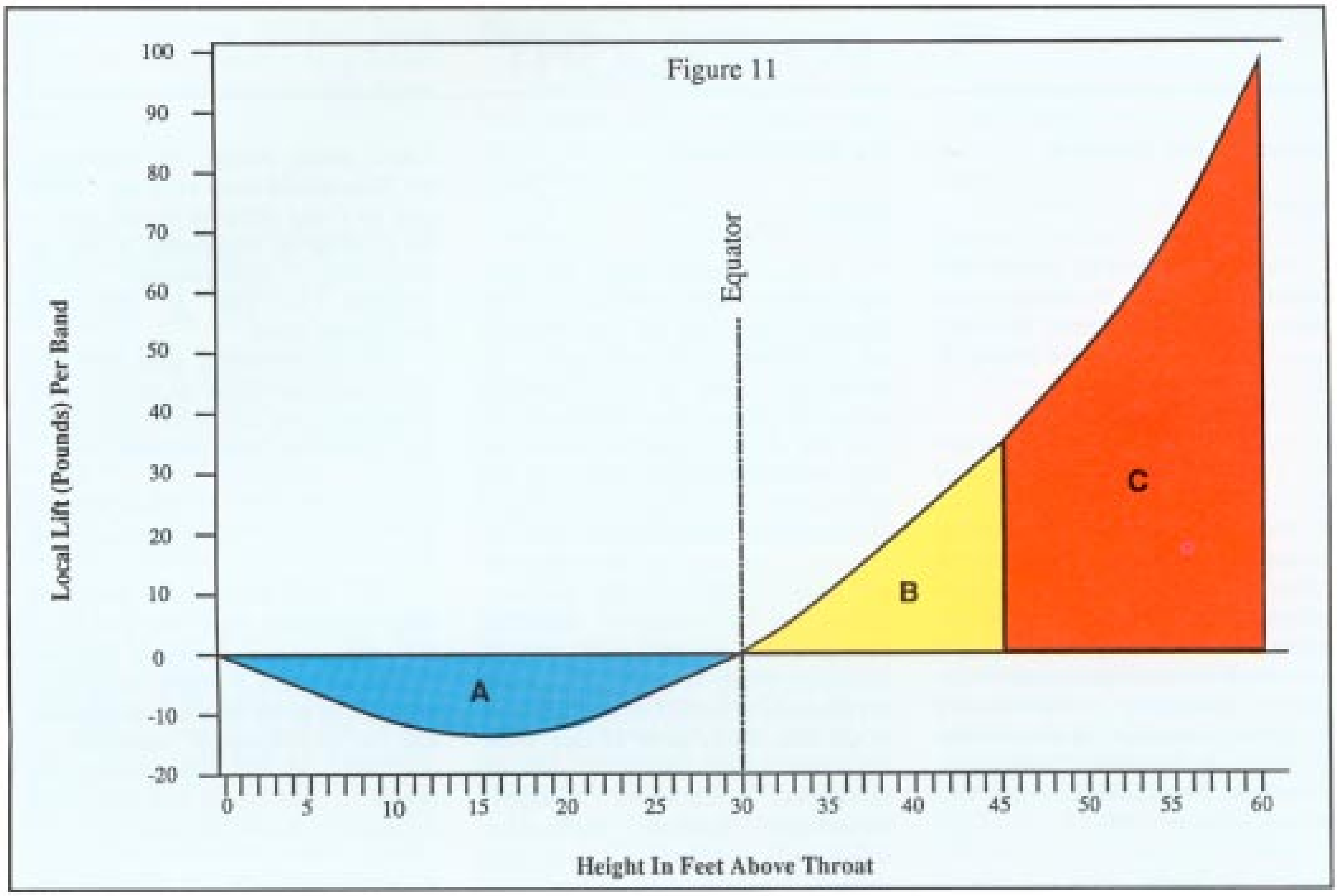


Figure 8

Figure 9







for the scope of this article.

To understand the curve in figure 10, let's start at the throat. Here the internal pressure of the balloon is the same as the external atmospheric pressure, the differential is zero. Therefore the forces near the throat are zero. As we move up the side of the balloon the pressure increases. However, all vertical components of the force point downward and oppose lift. These forces act as apparent extra weight! As we reach the equator we've got more pressure, but all vectors are pointing horizontally, i.e. no vertical components. Moving upward from the equator we begin to get positive lift since the pressure vectors start pointing slightly upward. Then as we move near the crown the vectors almost point straight up and the pressures in the balloon are at their highest.

So, there you have it! The way the forces behave to lift a balloon envelope. The curve in figure 10 was very interesting to me the first time that I generated it. It shows that the lower half of the balloon actually produces negative lift (equivalent to extra weight). When you see things like this you might ask: "Why not just get rid of the lower half of the envelope"? Well, if you did that, the pressure at the equator would be zero. Remember that the pressure starts building from the throat up. You need good pressure by the time the vectors start turning up just above the equator. Hence, the bottom of the balloon is needed to trap the air which creates the higher pressures in the top.

Going a step further, we can sum up the total increase in weight of the balloon due to down-sloping forces. This is equivalent to the shaded area between the curve and the zero local lift line (area "A" in figure 11). It turns out that this total downward force for a spherical balloon is exactly compensated by the upward force produced by the area of the balloon beginning at the equator and ending 3/4 the way up the balloon (area "B" figure 11). The next area (area "C" in figure 11) produces all of the usable lift.

Isn't that interesting though! What all of this means is that the balloon envelope, from the throat to a point 3/4 the height of the balloon, produces no net lift! When we get up the envelope 3/4 of the way to the crown only then do we start to get usable lifting forces.

Next, we might like to look at the magnitude of the forces in each of these regions. In working this calculation I made the assumption that I needed to lift 1000 pounds (gross load). For this case the downward force produced by area "A" is 250 pounds, the upward force produced by area "B" is 250 pounds, and area "C" generates 1000 pounds of residual lift.

By knowing how to think about these forces, those of you who like to do some figuring for home balloon designs can do some quick and dirty calculations to approximate the forces in the tops of your balloons. The first thing you have to do is figure out the surface area of your design that supports the load, i.e. where the vectors point up. Then determine how much load you want to carry, and divide the two, i.e. *LOAD* (pounds) divided by *AREA* (square feet) equals pressure (pounds/square foot). In most cases the result isn't quite exact for a couple of reasons. First, you're neglecting forces that don't point up, and second, you're assuming a uniform pressure distribution. Even if this technique isn't perfect you'll find, though, that it gives you a pretty good approximation.

Just for fun let's grind this calculation out for our spherical balloon. Since the top 1/4 of the envelope supports all of the weight and we have a 60-foot diameter, the surface area of that section is

$4\pi R^2$=Surface Area of a Sphere

$1/4(4)\pi R^2$=Area of top 1/4 of the Balloon where π=3.14, and R (radius) = 30 feet. So, the two 4's cancel each other and

=3.14 (30) (30) = 2826 square feet.

(lets call it 2800). If we need to lift 1000 pounds gross load then all 1000 pounds is distributed on this 2800 square feet of fabric which means we need a pressure differential of

1000 lb/2800 ft^2 = .36 lb/ft^2

or about 5.7 ounces per square foot. Because the pressure in the top isn't really constant the real values range from about 5 ounces per square foot at the 3/4 mark to about 7 ounces per square foot at the crown. As you can see, though, if you're just trying to get an idea of the forces (within a half pound or so), this method works real well.

Let's consider a different geometric example for those of you who might like to fly a can shape (Figure 12). For a good comparison let's build a balloon of the same height and cubic

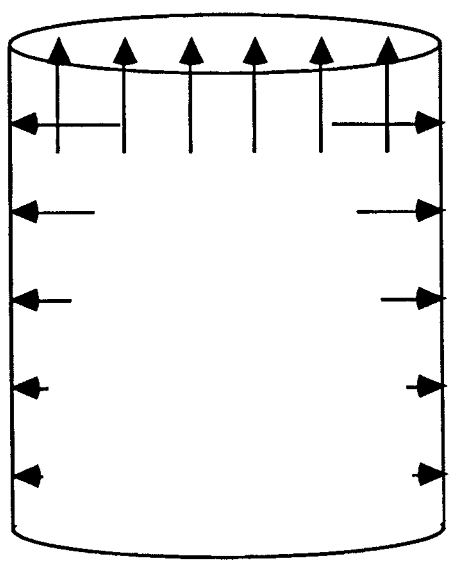


Figure 12

volume as our spherical example (113,000 cubic feet). To do this we make a cylinder 60 feet high and 49 feet across. Here the side vectors point horizontally all the way up the balloon. Only the top area makes lift to use for flight. A simple calculation shows the lifting area (the lid) to be 1883 square feet. If we want to carry the same 1000-pound load this fabric must support a pressure of

1000 lb/1883 ft^2 = .53 lb/ft^2 = 8.5 oz/ft^2

That's an increase of 2 to 3 ounces per square foot over a sphere. So you can see as we deviate from the spherical shape things can get worse. To fly this can, the pilot would have to run hotter to generate the higher pressures. This reduces fabric life and uses more fuel, but there is a price for everything. I personally like to see the fun new shapes and designs being built!

Epilogue

After my first article on *The Physics of Lift in Hot Air Ballooning* (*Balloon Life,* May 86) I received many letters from home designers and other very interesting people. That correspondence stimulated me to write this article.

It's wonderful to live in time when we can fly! Happy Heights!

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